Malaysian Journal of Economic Studies Vol. XXXIX Nos. 1 & 2, 2002 ISSN 1511-4554

On the Formulation of an Optimal Growth Programme of the Global Knowledge Economy: Experiments for Numerical Implementation of Endogenous Growth Models[1]

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Abstract: Existing optimal global models specify the elements of a national optimal growth programme. This paper presents an operational optimal growth model of the global knowledge economy. The modelling study focuses on the issues of optimal structures of endogenous and exogenous growth paths and the long-term dynamics of the global knowledge economy and its characteristics. The focus of the study is on the operationalisation of the model, rather than studying the detailed issues of endogenous technical progress, global development and the emergence of the knowledge economy.

1. Introduction

Advances in information technology, the emergence of the global knowledge economy and the high speed and intensive process of globalisation have made the study of the global economy a matter of great interest and an academic challenge (McKibbin and Sachs 1991; Sheehan and Tegart 1998). Numerical real life modelling of the global economy is currently of great interest. Works in this area have been well surveyed in Klein and Lo (1995) and Hickman (1983). The work by McKibbin and Sachs (1991) is probably the best text on global economic and growth modelling. Many of these studies are descriptive models of the global economy in a behavioural sense. Large-scale optimisation global economic growth models also exist. However, most of the optimal global growth models have been developed as descriptive models of the global market economy. The applications of the elements of a normative optimal growth model to optimal global economy modelling are neither well researched nor

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** This paper is based on Islam (2001b) published by North Holland Publishing. The author thanks Elsevier Science/ North Holland for their permission to prepare this paper on the basis of materials in Islam (2001b). The author wishes to thank Prof. W. Nordhaus, B. Craven and Prof. Dr. R. Guest for their extensive assistance in various forms in the undertaking of this study and to Jamie Sanderson and Kurt Brinschwitz for providing excellent research assistance.

[1] Seminal contributions in this area of global modelling have been made by Nordhaus (1994) and Manne (1985). This modelling work owes a lot to the DICE model of Nordhaus (1994). However, the present set of models is different from the DICE model. The present models are standard neoclassical and endogenous growth models. The DICE model has economic and climate change sectors while the present models have only economic sectors. In this version of the global model, an aggregate model of the global economy is developed. Several multi-regional versions of the model have also been developed by the present author including a version where Australia has been specified as a separate region (Islam 2001a; Islam 2001b). Other versions have also more detailed relationships of the financial sector of the global economy such as exchange rate, price level, nominal economic variables, capital flows, and money supply and demand.

developed. There is a need to investigate this area further. This can be done by developing normative optimal global growth models to characterise the structure and pattern of optimal growth of the global economy.

A relatively recent phenomenon in the global economy is the emergence of the knowledge economy. In the theoretical front, an advancement in explaining global economic issues is provided by endogenous growth theory (Romer 1986; 1990). It is possible to model the knowledge economy within the framework of endogenous growth theory. It is widely accepted that endogenous growth models provide additional insights into the understanding of the process of economic growth and generate improved understanding of growth economics of the developed western world. Mathematical modelling of the global knowledge economy is not well known. We argue in this paper that endogenous growth theory can provide a framework for modelling the global knowledge economy. However, although analytical endogenous growth modelling is well developed, operational modelling of endogenous growth theory as a programme for optimal growth has not developed much. There are some numerical studies of regional or national economies (Robertson 1995) but these studies have not highlighted the choice of an optimal endogenous growth programme in the same formal basis as it has been given in works like Chakravarty (1969).

The aim of the paper is to develop an approach for the specification and implementation of an optimal growth programme for the global knowledge economy which can overcome the limitations of the existing literature. The detailed objectives of this paper are:

1. To develop an optimal growth model of the global knowledge economy in an endogenous growth modelling framework and as an optimal growth programme
2. To make an assessment of the computational difficulties and opportunities in modelling the global knowledge economy
3. To use the developed model to determine the ideal or optimal structure of the global knowledge economy and predict its future
4. To investigate the properties of optimal growth dynamics of the global knowledge economy in terms of the mathematical properties of an optimal growth model.

The structure of the paper is as follows: Section 2 surveys the relevant economic issues and the justifications for the modelling methods adopted in this paper. A theoretical optimal growth programme for the global knowledge economy is specified within the framework of endogenous growth theory in Section 3. Section 4 provides the rationale for specification of an optimal growth model of the global knowledge economy. Computation of the model is discussed in Section 5. Conclusion and areas for further research are stated in Section 6.

2. Growth of the Global Knowledge Economy: Issues and Modelling Perspective

2.1 Issues in the Global Knowledge Economy

The growth rate of the global economy has been relatively fast with increasing availability of material goods, services, and wealth as well as knowledge. This has been possible due to increases in capital stock, knowledge, science and technology, population, innovation, the supply of resources, better management practices, improved productivity, and other related economic factors. The accelerated pace of global economic growth has changed the traditional structure and institutional basis of the global economy, accompanied by commonly cited by-products of fundamental consequences such as globalisation, the emergence of the knowledge economy, global environmental degradation, and regional convergences and divergences of economic growth of different countries (Sheehan and Tegart 1998). Some important characteristics of a knowledge economy are the increasing knowledge intensity of the economy, rise of the online economy, rising value of knowledge and market failures, and integrated international markets and globalisation (Sheehan and Tegart 1998).

With the emergence of the knowledge economy and the changes in the growth rate and structure of the global knowledge economy, several issues about its analysis and management have originated. Some of these issues, especially those relevant from an economic growth perspective, may be summarised as follows:

1. What are the economic growth implications of the emerging global knowledge economy for global economic growth and its structure?
2. Has the emergence of the knowledge economy led to convergence of growth rates among different economies with different levels of income?
3. Has globalisation, which is a result of the knowledge economy, as is sometimes argued, been favourable for increased economic growth and equity of income of different nations?
4. What is the prospect of the global knowledge economy growing along a stable, steady-state path indefinitely, without business cycles, depressions and regional disparities?

2.2 Modelling of the Knowledge Economy

Integrated framework and theories of global economic growth can be developed to address the emerging issues of the global knowledge economy. OECD (1992) has listed three schools of global growth theory: the equilibrium, coordination and free market perspective. There are many other views as well. Mathematical models can be developed on the basis of different ideological perspectives and can be used to analyse and forecast some of these issues, changes, and growth of the global economy. This model can generate scenarios in conformity with the theoretical underpinnings of the model used to generate them.

In a similar fashion, models for the global knowledge economy need to be developed. It is argued here that the characteristics of the knowledge economy can be incorporated into an economic model by an appropriate specification of technical progress functions. This can be accomplished by the specifications of: (i) the endogenous technical progress function; and (ii) a relatively higher rate of technical progress in the economy, a high proportion of GDP allocated to R&D and inventive activity, high research success coefficient, and a low rate of decay of technical knowledge in the technical progress function. This provides the prominence of knowledge or technology envisaged in the knowledge economy. The endogenous growth theory provides an appropriate framework for specifying the above characteristics of technical progress functions.

In the next section, we discuss some essential features of the endogenous growth theory which make it an appropriate framework.

3. Endogenous Growth Theory and an Optimal Growth Model of the Global Knowledge Economy

It is a common argument in economics that technical changes take place within an economy endogenously determined by socio-economic forces depending on the processes of invention, innovation, diffusion and market penetration (endogenous technical progress).

There are several arguments that we can put forward for the specification of technical progress endogenously in a growth model:

1. To incorporate the mechanism and the scope for government intervention in a growth model.
2. To explain the role of total factor productivity in the economic growth model, which is lacking in the neoclassical formulation of growth as defined by Solow (1956) and the Harrod-Domar growth model.
3. To include the possibility of increasing returns to scale in the growth formulation. In the neoclassical model decreasing returns to capital and labour are assumed. This relaxed assumption recognises that augmentation of capital and labour will boost returns.
4. To explain the observed behavioural issues that are not currently addressed by the neoclassical growth model, for example, the failure of the convergence hypothesis.

There are various ways in which endogenous technical progress can be included in the neoclassical growth model. Generally two broad approaches are taken.

The first involves avoiding the diminishing returns to scale in capital. Rebelo (1991) proposes the AK production function where K is a broad definition of capital while Barro (1990) proposes the inclusion of public and private capital into the production function, the sum of which will avoid diminishing returns to scale in capital.

The simplest representation of the endogenous growth theory may be provided in terms of the AK production function in which K is the total stock of capital including physical capital and human capital and A is the level of technical progress in the global economy of the following form (based on Gandolfo 1996):

$$Y = AK$$

where Y = gross output.

To demonstrate two essential properties of economic growth which are endogenous growth and convergence, the production function can be specified as follows:

$$Y = F(K,L) = AK + \Omega(K,L)$$

In this form of production function, the law of diminishing returns to capital is introduced, but a positive lower limit to it is specified unlike one of the Inada conditions where it is allowed to be zero.

A Cobb-Douglas specification of the production function is given as:

$$Y = F(K,L) = AK + BK^{\alpha}L^{1-\alpha}, \qquad 0 < \alpha < 1$$

and in per capita terms:

$$y = f(k) = AK + BK^{\alpha}$$

An endogenous optimal growth model of the global economy can now be presented in the following form:

$$\max_{\{c(t)\}} U = \int_0^{\infty} e^{-(\rho - n)t} u[c(t)]\,dt$$

where ρ = discount rate,
n = rate of population growth.

For the derivation of an optimal structure of the growth paths of the global economic variables, we form the Hamiltonian of the problem:

The list order of conditions for maximisation of the Hamiltonian provides the following steady state values of $c(t)$ and $k(t)$:

where θ is a constant elasticity of marginal utility.

If we assume that $A > \delta + \rho$, then both c and k grow without bound. Unlike neoclassical growth theory, the growth rate of these variables in an endogenous growth model is positive in the steady state. y, c, k all grow at the same rate (balanced growth).

The transitional dynamics of this endogenous growth model cannot be constructed in terms of the original variables of the model as the dynamic system generated from the optimisation process does not have a singular point. Normally the original variables are constant in the steady state and then it is shown that the endogenous growth model has a saddle point implying the possibility of a stable equilibrium. For the existence and globality of this equilibrium solution, the model may satisfy the general Weirstrass theorem (Intriligator 1971) which is not satisfied in most versions of endogenous growth theory.

The second approach pursues the redefinition of capital and in this fashion avoids the diminishing returns to scale in capital. Romer (1986) and Arrow (1962) propose that 'knowledge' is generated in the production process and thus the 'state of knowledge' should be included in the production function. Lucas (1988) suggests that labour (human capital) is generated in the production process in a constant return to scale production function. The labour generates the human capital from 'knowledge' which feeds back into the production process avoiding the decreasing return to capital and contributions to growth. Barro and Sala-i-Martin (1995) argue that intermediate goods are produced and converted to final goods via the application of R&D activity. There are constant returns to scale in the variety and number of intermediate goods and decreasing returns to scale in the production of intermediate goods. The balance between these two situations is controlled by the subdivision of the R&D increases in output due to decreasing returns to scale.

From the above survey, it will appear that endogenous growth theory has several features such as the possibility of increasing returns, and the prominent role of technology, knowledge, information, product variety, public investment and policy which can represent the characteristics of the global knowledge economy discussed previously.

The processes of invention, innovation and diffusion and their effects on globalisation (Coe and Helpman 1995; Feenstra 1998; Jovanovic and Rafael 1989) are incorporated in the model by giving higher values of the coefficients of technical progress function showing the greater importance of technical progress and its role in enhancing globalisation.

This optimal growth model of the global knowledge economy has the following elements of an optimal growth programme (Burmeister and Dobell 1970; Chakravarty 1969; Fox *et al*. 1973): (i) the length of the planning horizon; (ii) the choice of an optimality criterion; (iii) the rate of time preference; (iv) the terminal or transversality conditions; and (v) an economic model.

4. The Optimal Global Growth Model

The central issue in the determination of an optimal growth programme of the global knowledge economy is the rate at which the global community should save, invest, spend for knowledge accumulation and consume out of the global income so that the global economy grows at the optimal growth rate (i.e. the optimal rate of global economic expansion and capital accumulation) over an infinite or finite time horizon. An optimal growth model of the global knowledge economy is specified to identify an ideal system of intertemporal resource allocation in the global economy and should be developed within the framework of the principles, methods and theories of optimal economic growth.

The choice of the elements for an optimal growth programme is important as there are several options related to each of them and choosing a particular specification affects the nature of the optimal growth programme significantly. Following the literature on the specification of the elements of an optimal growth model of a national economy, the dominant practice is to adopt a similar set of elements of national optimal growth models in global models without discussing the relevance of the elements of national models in global models. In the specification of the elements of an optimal global growth model of the global knowledge economy, however, the following issues need to be considered.

1. Can a utility maximisation objective of a national model be appropriate for an optimal growth programme of the global knowledge economy?
2. Are the procedures for the determination of the social discount rate (SDR) of a national economy equally applicable to the specification of a global discount rate?
3. In different economies with different economic organisations and outlooks, how can we determine the planning horizon of an optimal global growth programme?
4. As different regions will grow and want to grow at different rates, how can we determine the terminal conditions of the global model?
5. Given historical evidence of non-convergence in global growth and non-convexity of an endogenous growth model, the proof of the existence of general equilibrium in the global economy by the fixed point theorems may become problematic. In these cases of non-convexity of the opportunity set, can we still specify the optimal growth model in its equilibrium form or will some other forms of disequilibrium analysis be more relevant (see Thompson and Thore 1992)?
6. What type of model can appropriately represent the structural and institutional relations of the global knowledge economy?
7. Can a global knowledge economy model represent a state of efficient allocation of resources

irrespective of the underlying structural and institutional relations of the economy assumed in the model?

The above issues provide the need for the specification of the elements of an optimal global model differently from the practices of the specifications of elements of a national growth model. This point is not well stressed in the existing global models.

4.1 Discrete Modelling: An Empirical Endogenous Growth Optimisation Model of the Global Economy

First, we present the computational experiments with a small discrete optimal global growth model named Endogenous Global Optimal Growth (EGOG) model developed in this study. In Section 5.4, computational methods for continuous optimal global growth models will be discussed. The EGOG model is an operational aggregative growth model of the global economy which includes technical progress that is to be determined endogenously and is specified within the context of the elements of an optimal growth programme. This is a social planning model of the global knowledge economy.

The EGOG model is similar to standard optimal economic growth models of the national economy. This numerical model is based on the work done previously in this area by Chakravarty (1969), Kendrick and Taylor (1971), Manne (1985) and Nordhaus (1994), among others. The social planner optimises social utility over a given time horizon subject to the economic constraints of the capital accumulation equation, production function, income composition equation and endogenous technical progress function. Social utility is defined as the sum of individual utilities and based on a broad concept of cardinal utility of consumption and specified in the logarithmic form with constant elasticity of marginal utility.

The economy produces one composite good, which is optimally allocated by the social planner between consumption, investment and technical progress (R&D etc.) to maximise social utility, while population growth is assumed to be exogenous. The social planner allocates resources to augment technical progress in the economy for higher productivity of factors of production and a higher GDP growth rate, leading to higher levels of consumption and social welfare.

Although the present model is developed as a central planning model, following the well established practice in economics (Fox *et al.* 1973) the shadow prices of capital, labour, R&D can be adopted to replicate the technology market in the economy.

In the present model, the specification of the elements of the optimal growth programme was relatively easy since the model is an aggregative model. Following the mainstream practices in this area and the arguments stated in Islam (2001b), the model is specified by embedding a set of elements of an optimal growth programme of the following forms: an utilitarian optimality criteria, an endogenous growth model, finite planning horizon, positive time preference, and boundary conditions specified on the basis of the historical conditions in the global economy. The model here can represent the structural and institutional relations of the global knowledge economy in a very simple form. The above specification of the elements is not consistent with the economic and institutional characteristics of the global knowledge economy.

A presentation of the EGOG model with an endogenous technical progress function is given below. The model maximises the sum of discounted social utilities. The objective function is

The explicit form of the utility function is assumed to have the CES specification, hence

and $U[c(t), L(t)] = L(t) \log c(t)$ when $\alpha = 1$

The production function is the standard Cobb-Douglas specification with labour and capital as factor inputs and with endogenous technological change[2]

The following relation gives the endogenous technical progress function and the technical progress time path (alternative forms are discussed in the next section):

$$A(t) = A(D(t), A(t\text{-}1), g_A(t), g_A(t\text{-}1), \delta_A)$$

The growth rate of labour input is assumed to decay exponentially so that the rate reaches an asymptotic level. The time path of the growth rate of the variables is

$$g_{pop}(t) = g_{pop}(t\text{-}1)(1\text{-}\delta_{pop})$$

The flow equation for the capital stock $K(t)$ is the usual investment relationship

$$K(t) = (1\text{-}\delta_K)K(t\text{-}1) + I(t\text{-}1)$$

The social planner allocates output into consumption $C(t)$ or investment $I(t)$

$$Q(t) = C(t) + I(t) + D(t)$$

Terminal constraint on capital is:

$$(1+\rho)^{-T}K(T) = I(T) \qquad (1)$$

The model also calculates some major rates and ratios in the economy (such as the saving, wage and interest rates, etc.) in the following forms:

$S(t) = I(t)/Y(t)$	-	Saving rate
$W(t) = (1\text{-}\gamma)\ Y(t)/L(t)$	-	Wage rate/share
$R(t) = \gamma Y(t)/K(t)(1\text{-}\delta_k)$	-	Rate of interest
$Y(t)/C(t)$	-	Consumption-GDP ratio
$K(t)/Y(t)$	-	Capital-output ratio
$K(t)/L(t)$	-	Capital-labour ratio

2 It is assumed that labour inputs are proportional to population and no unemployment exists in the economy.

where:

t	=	time (1, 2, ..., T)
$c(t)$	=	per capita consumption
$A(t)$	=	technical progress
$K(t)$	=	capital stock
$L(t)$	=	population labour supply
$C(t)$	=	consumption
ρ	=	discount factor
α	=	elasticity of marginal utility of consumption
κ	=	terminal period capital coefficient
$S(t)$	=	saving rate
$Y(t)$	=	GDP
$I(t)$	=	investment
$W(t)$	=	wage rate/share
$R(t)$	=	interest rate
$D(t)$	=	research and development
$g_A(t)$	=	growth rate of technology
d_A	=	decline rate of technology
γ	=	elasticity of output with respect to capital
$(1 - \gamma)$	=	elasticity of output with respect to labour
δ_k	=	capital depreciation rate
g_{pop}	=	growth rate of population
δ_{pop}	=	decline rate of population.

4.2 Alternative Specifications of Endogenous Growth/Knowledge Economic Equations

Starting with the work of Kaldor (1957) on a technical progress function, different technical progress functions have been specified in growth models. Some of the well known endogenous technical progress (A) functions (TCF) compiled from different studies are as follows.

$A = f(X)$	X	=	Share of investment in GDP	
$Q = AK$	Q	=	Output, K = Capital including human capital	
$A = e^{rt} G^{\alpha}$	G	=	Public expenditure	
$A = e^{rt} H^{\alpha}$	H	=	Human capital	
$A = e^{rt} D^{\alpha}$	D	=	Research and development expenditure	
$A = f(\lambda , A_{t-1})$	λ	=	Share of research expenditure in GDP	(2)
$A = f(P_i /P_j)$	P_{ij}	=	Prices of input i and j	
$A = s^{\xi}$	s	=	Level of investment in research, ξ = Exponent	
$A = f(A_{t-1} , L_R / L)$	L_R	=	Labour allocated to the research sector	
	L	=	Total labour	
$A = f(P_j)$	P_j	=	Index of product variety	

Four commonly used technological change functions were adopted in this model. These functions and the values of different parameters in these functions are discussed below.

The first is a first order forward discretisation of the TCF in the form of a differential equation formulated by Shell (1966) which is given in equation (3). The Shell model of technological change makes the TCF dependent on the amount of resources devoted to inventive activity and the depreciation rate of technical knowledge. Where α is the fraction of output allocated to inventive activity at time t, β is the rate of decay of technological change and σ is the research success coefficient, the TCF of Shell can be specified as follows:

$$A_{t+1} - A_t = \alpha\sigma Y_t - \beta A_t \qquad (3)$$

The values of the parameters were based on historical data available in this area and collected from different published sources. The values used were:

$\alpha = 0.015$
$\sigma = 0.285$
$\beta = 0.011$

The R&D activity was considered in the formulation by altering consumption to be net of R&D investment, as follows:

$$c_t = (1 - \alpha)Y_t - I_t$$

The second specification is a second order discretisation of the differential equation A_t:

This specification was used in a forecasting model (Islam *et al.* 1997) to determine the parameters determining technological progress. The model parameters were extracted from the exercise by making the forecasting simulation emulate growth paths of the various variables equivalent to the growth paths of the economic variables as determined in the DICE model of Nordhaus (1994). The estimated parameters of the equation were then used as input to this model to simulate the results.

The initial conditions of the forecasting were as follows:

The TCF parameters were determined to be:

$\beta = 0.075$
$\gamma = 0.025$
$\sigma = 0.01$

In the third specification of the TCF, the TCF grows exponentially and is a function of the saving to GDP ratio, s_t:

Here the saving rate, s_t is defined as the ratio of investment to output.

In the fourth formulation of TCF, the production function that incorporates a human labour component and the human capital equations were used. The system of equations to represent the human capital and the consequent production functions may be stated as follows (Robertson 1995):

The first equation H_t represents a human capital evolution; the second is the initial condition; and the third is the modified production function which incorporates human capital. The control variable u_t represents the proportion of time spent by an individual on the production of goods; conversely the term $(1 - u_t)$ represents the proportion of time spent upon the acquisition of new skills, as opposed to that spent in actually producing goods.

5. Methodology and Results

The EGOG model was solved by the GAMS programme (Brooke *et al.* 1997) which implements the gradient search algorithm implemented by MINOS as a discrete optimal control problem (a discrete dynamic optimisation problem). The approach is based on a nested two-level algorithm, an inner simplex method and non-linear-programming algorithms combining a reduced gradient and quasi-Newton method with a projected Lagrangean algorithm as in Brooke *et al.* (1997).

Data for the model, except the parameters of the endogenous technical progress function discussed above, was adopted from Nordhaus (1994) which contains discussions about the sources of data and the justification for their uses in the model. Table 1 contains the major aggregate and compiled data used in this model.

For economic reasons a pure time preference of 3 per cent was used in this model, the specification of the transversality and boundary conditions given in Table 1 were specified in this model to attain a convergent solution.

5.1 Computational Experiments

The EGOG model was solved for four types of specification of the endogenous technical progress function discussed in Section 4.2.

The model with the first TCF (equation 3) converges extremely slowly compared to the model with exogenous technical progress, with the overall economy growing slowly initially, but increasing at an exponential rate towards the end of the planning horizon. The resulting global GDP values shown in the results clearly demonstrate this behaviour.

In the case of the second TCF, as in the case of the first TCF, the economy was growing slowly. An extra constraint was added to force technical progress to grow more quickly. The constraint is shown below with Δ_t being the minimum rate of change per decade and is determined from data values given by Nordhaus (1994: 13).

The addition of this equation caused the model to become infeasible.

We were unable to solve the formulation of the model with the third TCF as the exponential formulation lead to numerical instabilities and an unbounded solution. Enforcing bounds $0 \le s_t \le 1$ did not help to obtain an acceptable solution. It is suspected that the extreme non-linearity of this formulation of technical progress leads to numerical instabilities and an unbounded solution which the optimisation routines could not adequately deal with.

The models with the fourth TCF converge, with some interesting characteristics, as there was a decline in production and the general economic variables for the first five or so decades.

Table 1: Data, boundary and transversality conditions

Parameters	Run 1	Run 2
Elasticity of marginal utility	0.00	0.00
Rate of social time preference per year	3.00	3.00
Growth rate of population per year	2.33	2.33
Decline rate of population growth per year	0.0195	0.0195
Depreciation rate of capital per year	0.10	0.10
Growth rate of technology per year	0	1.5
Decline rate of technology growth per decade	0	0.11
Capital elasticity in output	0.25	0.25
1965 gross global output (trillion 1989 USD)	8.519	8.519
1965 global population (millions)	3369	3369
1965 value capital (billions 1989 USD)	16.03	16.03
Initial level of total factor productivity	0.00963	0.00963
Transversality capital coefficient	340	340

Economic growth then improved, but did not grow as rapidly as for the exogenous model, which we used as the base case for comparisons.

The difficulties encountered in the numerical implementation of an optimal growth model with endogenous technical progress is not unique to this study. Chaudhury (1989) has a summary of the effects of technical progress in terms of the growth rate, instability and stagnation in different growth models. In an endogenous growth model when technical progress is endogenous, GDP growth may be capped (Sato 1966; Chiang 1992: 269), growth paths may become unstable and explosive and any optimum solution may not exist at all (Barro and Sala-i-Martin 1995; Gandolfo 1996).

These difficulties are probably caused by the additional endogenous variable, endogenous technical progress in the optimisation model which causes the feasible region to be relatively more restrictive or by the possible non-convexity of the endogenous technical progress function or of the knowledge economic model. However, further research is needed to determine the exact causes of the difficulties of a numerical endogenous growth model.

If a dynamic optimisation model satisfies the generalised Weierstrass condition, the GAMS programme can find a unique global optimum. However, most of the economic dynamic optimisation models are non-convex. Two types of problems can arise in these models: no solution may exist or only a local solution may exist. There are several approaches that we can pursue in this situation: (i) accept the local solution as a reasonable solution; (ii) adopt some other computational procedures; (iii) respecify the problem; or (iv) resolve the model with an alternative set of initial and terminal conditions to examine whether the local optimum is reasonably robust or not.

There are several computational approaches to deal with non-convex optimisation problems. These methods are classified as global optimisation methods and they include computation procedures such as parametric programming search methods, table search, simulated annealing, parallel architecture, genetic algorithm etc. (Kendrick *et al.* 1991).

Because of the difficulties encountered in solving the different versions of the EGOG model, only results of the model with the first TCF (equation 3) are reported in this paper. The present model incorporates different formulations of endogenous growth models including the research and development model of Romer (1990). However, in the computational experiments in the present work, only the endogenous technical progress function in equation 3 is numerically implementable. The features of the endogenous technical progress equations are similar to those of Romer's formulation of endogenous technical progress function.

The model was also solved for the case of exogenous technical progress. As we can see, Table 2 contains the results of two runs of the EGOG model in cases of endogenous and exogenous technical progress. Model Run 1 is the solution with endogenous technical progress, while Model Run 2 simulates the model with exogenous technical progress. The form and coefficients of exogenous technical progress (Kendrick and Taylor 1971; Nordhaus 1994; Shell 1966; Chakravarty 1969) are given below:

$g_A(t) = g_A(t\text{-}1)(1\text{-}\delta_A)$ - Growth rate of technology
$A(t) = A(0)e^{g_A(t)}$ - Growth of technology

where:
$A(t)$ = technical progress
$g_A(t)$ = growth rate of technology
d_A = decline rate of technology.

5.2 Growth Pattern of the Global Knowledge Economy

The optimum solution generated the growth path of the variables and parameters of the global economy over the next 50 years. Several factors including public expenditure and factors of production contribute to the production of GDP along with technical progress which is endogenously determined optimally by these factors. A general characteristic of the results of the model with the endogenous technical progress reported is that the growth paths are steady and positively increasing.

The model that incorporates exogenous technological progress shows a stronger positive trend in GDP but the level of GDP achieved in this model after 50 years is much higher than the endogenous model. Over the study period, Model Run 2 shows that GDP will rise from USD8.52 trillion in 1965 to USD31.17 trillion in 2015, while these figures are USD8.52 trillion and USD18.38 trillion respectively in Model Run 1 (see Table 2).

Associated with this lower growth rate of GDP in endogenous model is the lower growth rate in per capita income and consumption. Per capita income under an endogenous model rises from USD2,530 in 1965 to USD2,940 in 2015 while per capita consumption rises from USD1,970 in 1965 to USD2,390 in 2015 (Table 2). Growth rates of other variables and parameters are also higher in the exogenous model compared to the endogenous model. The model with endogenous R&D expenditure generates a decrease in economic activities since a share of GDP is allocated in this model for research and expenditure or financing technical progress. This leaves a lesser amount for investments resulting in relatively slower economic growth.

With the slower GDP growth, the optimal rates of consumption saving and investment in the endogenous growth model are forced to decrease sharply as the total amount of investment required to sustain growth is lower in the endogenous model.

Table 3 shows the comparative growth rates of the optimum values of different variables under the endogenous and exogenous models. The endogenous model shows slower growth rates in all economic variables.

Table 2: Dynamic paths for some economic variables in the models

	Model	1965	1975	1985	1995	2005
		Period				
Output (USD trillion)	1	8.521	11.200	13.713	16.105	18.377
	2	8.521	12.687	17.908	24.111	31.168
Consumption (USD trillion)	1	6.609	8.865	10.995	13.031	14.973
	2	6.649	10.018	14.280	19.383	25.223
Capital stock (USD trillion)	1	16.030	24.711	31.965	38.320	44.098
	2	16.030	24.309	35.173	48.546	64.212
Saving rate (fraction of GWP)	1	0.224	0.208	0.198	0.191	0.185
	2	0.220	0.210	0.203	0.196	0.191
Per capita income (USD 1000)	1	2.529	2.715	2.813	2.881	2.936
	2	2.529	3.075	3.674	4.313	4.980
Per capita consumption (USD 1000)	1	1.962	2.149	2.256	2.331	2.392
	2	1.974	2.428	2.930	3.467	4.030
Interest rate per annum	1	0.068	0.048	0.042	0.040	0.039
	2	0.068	0.065	0.062	0.059	0.056
Investment (USD trillion)	1	1.912	2.335	2.717	3.074	3.405
	2	1.872	2.670	3.628	4.728	5.945
Global shadow price of capital	1	926.941	629.620	446.254	321.365	233.048
	2	-176258	-209231	-241703	-273356	-303996

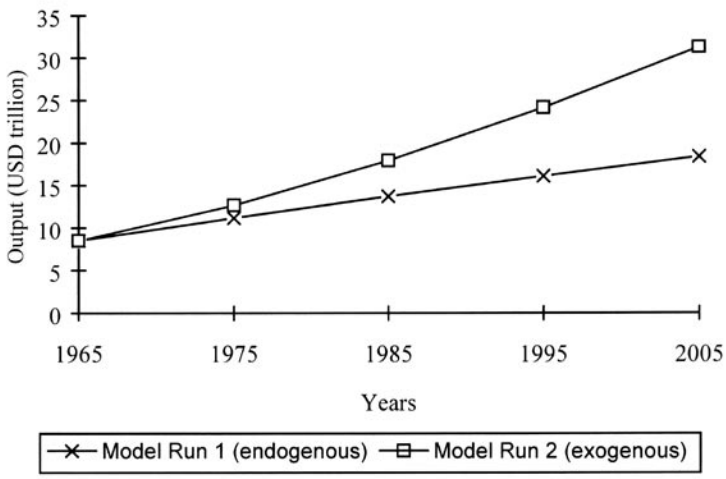

Figure 1: Growth of output over time

The results of slower growth in the endogenous growth model are inconsistent with the proposition of unbounded growth of endogenous growth theory. The results of relatively slower economic growth in this study are also not unique to this study. In the theoretical literature this possibility (that growth can be capped by endogenous technical progress) has been shown by Sato (1966). One possible explanation is that while technological progress is a free good in the case of exogenous technical progress, it is necessary to specify that a share of GDP is necessary to allocate for R&D in endogenous growth models.

5.3 Possibility of Steady, Stable and Unique Growth of the Global Knowledge Economy

In addition to generating the optimal paths of the global knowledge economy, the solution of a global growth model generates results that are useful in analysing the economic properties of growth dynamics of the global economy. The mathematical properties of any optimal growth model, such as the issues of existence, uniqueness, stability and other properties of a steady state solution and transitional dynamic paths of the variables and coefficients of the model (Ramanathan 1985; Intriligator 1971; Carlberg 1997), are of fundamental interest.

The results of EGOG are not broadly in conformity with the properties of the neoclassical type global growth model implying that the growth process in the global economy is characterised by a smooth, stable, limitless growth path, since we have experienced infeasibility and unboundedness. The global growth trajectories do not show a steady state within the specified planning period and experiments with longer time horizons did not produce a steady state either. The EGOG model was not solved for its steady state values. However, it demonstrates the turnpike property as the initial phase of the optimal paths of the economy did not change following substantial changes in the specification of the terminal capital stock.

Table 3: Average growth rates of selected variables (1965-2015)

Model Type	Output	Consumption	Per Capita Consumption	Investment
Endogenous	2.89	3.16	0.54	1.95
Exogenous	6.64	6.98	2.60	5.44

5.4 Continuous Modelling of Optimal Growth of the Global Knowledge Economy

Numerical implementation of a continuous optimal growth model is relatively difficult. Algorithms for optimal control (Teo *et al.* 1991) are based on several approaches such as: (i) dynamic programming; (ii) solving first-order necessary conditions of the Pontryagin theory; (iii) applying some approximation methods (steady state solution, numerical methods based on approximation and perturbation, and method of simulation); (iv) approximating the control; and (v) applying mathematical programming algorithms to a discretised version of the control problem.

In a recent paper (Islam and Craven 2001), the authors have discussed the use of optimal control methods for computing non-linear continuous optimal growth models. They have surveyed various recently developed algorithms for computing optimal control, involving step-function approximations, Runge-Kutta solutions of differential equations, and have suggested that the discretisation approach is preferable to methods that solve first-order optimality conditions. In that paper, they have reviewed some powerful computer programmes including MATLAB-RIOTS, MISER, and OCIM for computing such models numerically. A computer programme named SCOM by MATLAB-CONSTR was also developed in that paper.

While some computer packages are available for optimal control problems, they are not always suitable for particular optimal growth models, especially for computing optimal growth models with the elements of the knowledge economy which makes the model non-convex. Computation of a non-convex continuous optimal control problem such as an optimal growth model of the global knowledge economy is inherently difficult. The general alternatives stated in this paper are useful in a limited way. The gradient descent search methods are only effective if the model is not highly non-convex and if the local optimum is not very removed from the global optimum. Lipschitz conditions can be applied, but the problem of how to obtain the Lipschitz constant in the search method remains.

6. Conclusion and Further Research

From an operational point of view, the endogenous models specified in this study did not produce sufficiently encouraging results. In some of these models any feasible or optimum solution did not exist and some produced instability. The author knows some other examples of the study of operational optimisation modelling of endogenous growth theory (such as Robertson 1995), but they do not make any comparison of their results with exogenous growth modelling results. Therefore, the experience and results of this study cannot be compared with others in order to make an evaluation of the present study. A similar study of endogenous growth theory and climate change is reported in Islam (2001b).

The results show that the endogenous model produces a slower rise in GDP and consumption with a substantially slower level of investment and saving in the global knowledge economy compared to the exogenous model. This is in the opposite direction of what the endogenous growth theorists wanted to establish.

However, it needs to be emphasised that numerical endogenous growth knowledge economy modelling, especially in an optimisation framework, is still in its infancy. Experience gained in this study can be useful for other large-scale real-life global modelling studies. Further research in this area is necessary.

Some important areas for further research are as follows. Non-convex optimisation methods should be adopted to model the global knowledge economy. Models of multi-regional, multi-sector, multi-variable optimal global growth of the global knowledge economy can be developed with extended sets of economic relationships. Possible extensions which can be made in the present model are the inclusion of an input-output model in an optimisation framework, specifying constraints reflecting the economic conditions in developing countries such as the limited supply of capital goods, natural resources or foreign exchange limits, or the capital absorptive capacity. The objective functions can also be specified in different forms and include a different set of objectives such as employment, the balance of payments, inflation, stability, minimum basic needs, etc.

Uncertainty is a dominating feature in the long-term global knowledge economy. Stochastic optimal growth modelling (see Islam 2001b) of the global knowledge economy should be another interesting area for further research.

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