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ANALYSIS OF CONCRETE BEAMS WITH CIRCULAR WEB OPENINGS USING STRUT-AND-TIE MODELS

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Abstract: This study examines the strut-and-tie model for the analysis of reinforced concrete beams that contain geometric discontinuities in the form of a transverse circular opening in the web. Three T-beams with circular openings tested earlier by the authors have been chosen, and a strut-and-tie model was constructed for each beam. A comparison of the theoretical predictions concerning the ultimate strength, mode of failure and the proportion of applied shear carried by the chord members above and below the opening show good agreement with test results. The truss model also explains clearly the role of diagonal reinforcement in relieving concrete distress at the throat section by transferring a significant amount of applied shear across the discontinuity.

Keywords: *Cracking; Failure Modes; Reinforced Concrete.*

Abstrak: Kajian ini meneliti model tupang-dan-ikat untuk analisis rasuk konkrit bertetulang yang mempunyai ketakeselanjaran geometri dalam bentuk bukaan bulat melintang di dalam web. Tiga rasuk T dengan bukaan bulat yang telah diuji oleh penulis sebelum ini dipilih dan model tupang-dan-ikat dibina untuk setiap rasuk. Keputusan ramalan berdasarkan teori untuk kekuatan muktamad, mod kegagalan dan kadar ricih kenaan yang ditanggung oleh anggota pada bahagian atas dan bawah bukaan menunjukkan perbezaan kecil degan keputusan ujikaji. Model kekuda juga menunjukkan dengan jelas kepentingan tetulang condong yang membebaskan tegasan konkrit pada keratan leher menerusi pemindahan sejumlah ricih kenaan di sepanjang ketakselanjaran.

Katakunci: *Retak; Mod Kegagalan; Konkrit Tertulang.*

1.0 Introduction

In the design of concrete structures, it is necessary to distinguish between two regions, namely, the main regions and the local regions. In the main regions, often denoted as B-regions, stresses and strains are distributed so regularly that they can be easily expressed mathematically. That is, in these regions, stresses and strains are governed by simple equilibrium and compatibility conditions.

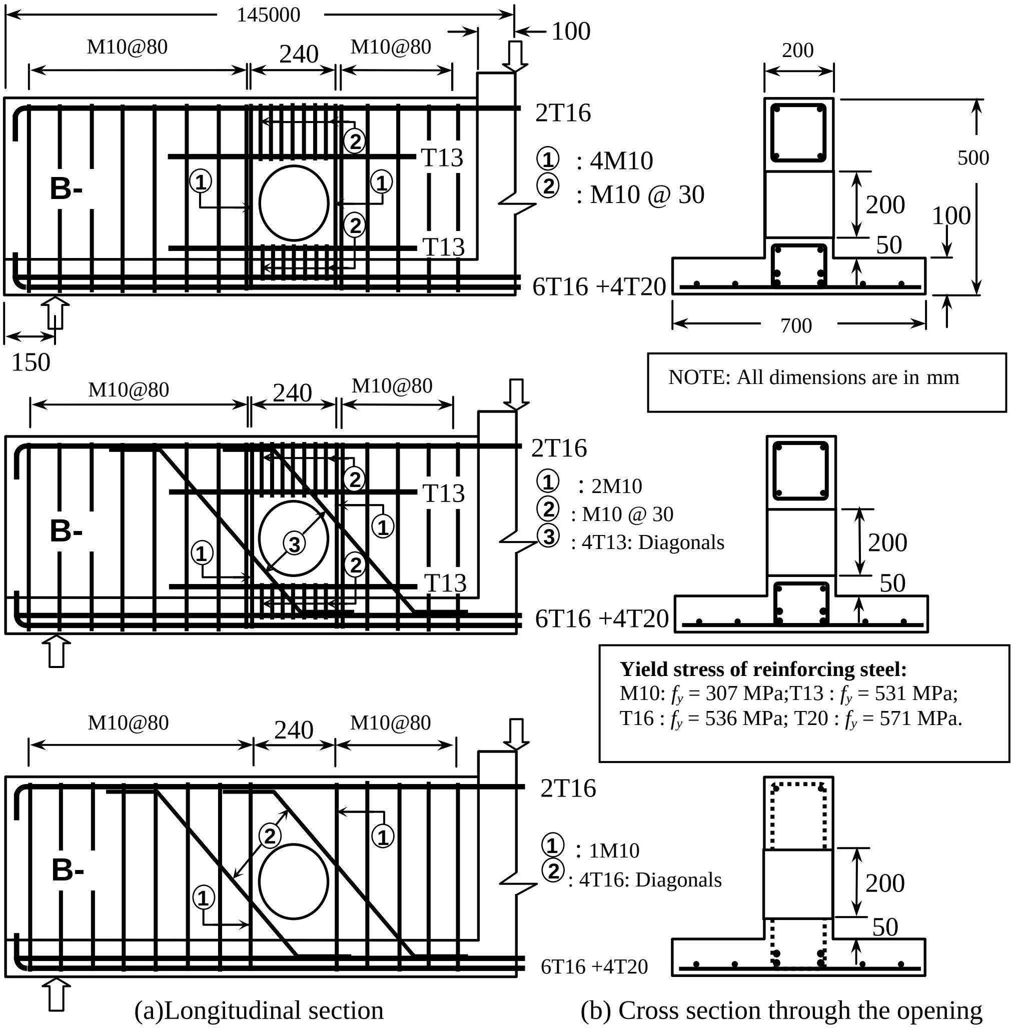
In contrast, the stresses and strains in a local region (denoted as D-regions), such as the ends of a beam or a column, the beam-column connections, the region adjacent to a concentrated load or a transverse opening, are so disturbed and irregular that they are not amenable to mathematical formulation using the basic requirements of equilibrium, compatibility and material laws. As a result, design of a D-region is usually based on simplified modelling using equilibrium conditions alone; the strain conditions in the composite (both steel and concrete) are neglected. One such model is based on the assumption that in a cracked reinforced concrete member, concrete in between the cracks carries direct compression and steel carries axial tension. The load-carrying mechanism of the member can then be idealised as that of a truss comprising a series of concrete struts and steel ties, and is known as ‘strut-and-tie model’ (MacGregor, 1998).

The present study is aimed at examining the adequacy of such a model in the perspective of three prototype reinforced concrete T-beams containing transverse circular openings tested earlier by the authors (Tan et al., 2001). Based on the reported cracking patterns and measured strains in the embedded steel reinforcement, a strut-and-tie model has been constructed and analysed for each beam. In this paper, the results of these analyses are presented and discussed from the viewpoints of ultimate strength, mode of failure and the proportion of the applied shear carried by the chord members above and below the opening.

2.0 Beam Details

Figure 1 shows details of the three beams R-4, R-5 and R-6, re-designated here as B-1, B-2 and B-3, respectively. Being reported earlier by the authors (Tan et al. 2001), these beams were identical in every respect, except for the amount and arrangement of reinforcement around the opening as shown.

All beams were 2.9 m long and contained a central stub to represent the interior support of a continuous beam. The cross section consisted of a 400 mm-deep and 200 mm wide web and, a 100 mm-thick and 700 mm-wide flange. A circular opening, 200 mm in diameter, was pre-formed in the web on each side of the central stub. It was located 50 mm away from the flange with its centre at 625 mm from the stub. The beams were identical with respect to the basic reinforcement, but differed in the amount and arrangement of reinforcement around the opening, as can be seen in Figure 1. They were tested in an inverted position under a point load applied to the central stub to simulate the conditions that exist in the negative moment region of a continuous beam. The full details of fabrication, test set-up, instrumentation and the test procedure followed can be found elsewhere (Tan et al., 2001).



NOTE: All dimensions are in mm

Yield stress of reinforcing steel:
M10: f_y = 307 MPa;T13 : f_y = 531 MPa;
T16 : f_y = 536 MPa; T20 : f_y = 571 MPa.

(a)Longitudinal section (b) Cross section through the opening

Figure 1: Beam and reinforcement details.

3.0 Summary of Test Results

The results generated from the testing of the three beams have been reported and discussed in detail in an earlier paper (Tan et al., 2001). For ready reference, a summary of the major results together with some additional information relevant to the main theme of this paper are included in Table 1 and Figures 2 and 3.

It may be seen in Table 1 that despite all the beams containing the same amount and arrangement of the basic reinforcement, beam B-1 demonstrated early cracking and displayed the widest crack width at the calculated service load (Test ultimate load/1.7).

Table 1: Summary of test results

Beam	Cylinder compressive strength f_c' (MPa)	Cracking load		Maximum crack width at service load* (mm)	Ultimate load (kN)	Shear carried by the top chord (kN)	Mode of failure+
		Flexural crack (kN)	Shear crack (kN)				
B-1	33.6	125	100	0.54	669.8	234.4	WC
B-2	37.2	130	160	0.36	935.8	257.3	F
B-3	37.2	130	150	0.32	921.2	216.5	F

Notes * Service load was taken as experimental collapse load/1.7.
+WC: failure by crushing of the concrete in the compression chord; F: Flexural failure.

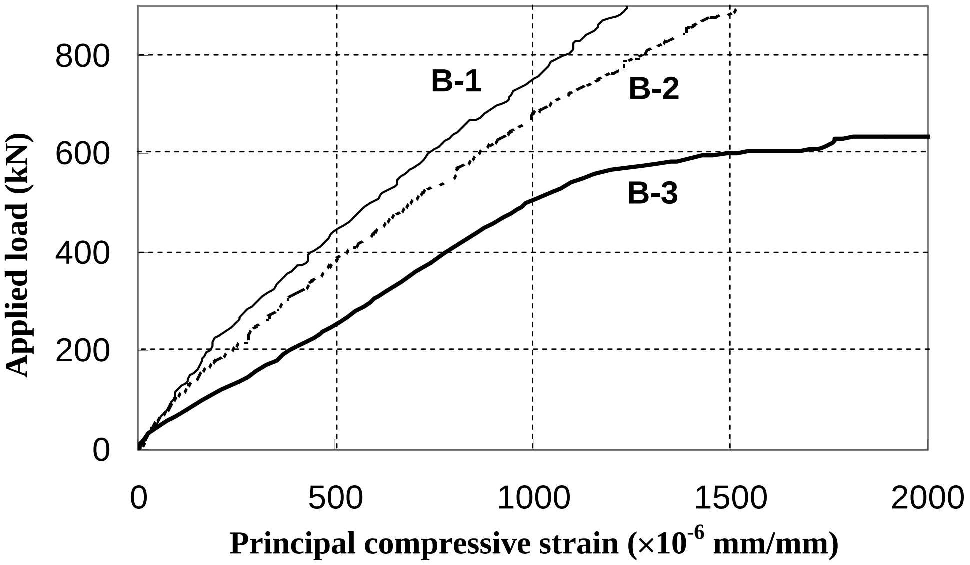


Figure 2: Load vs. principal compressive strain for concrete in the compression chord.

Also, this beam failed at a load much smaller than that of beams B-2 and B-3, and this occurred by crushing of the concrete in the compression chord (above the opening). This calls for paying careful attention in detailing the reinforcement around the opening region. The principal compressive strains in the concrete at mid-depth of the compression chord, that were measured by using a strain rosette, have been extracted from the thesis by Weng (1998) and are plotted against the applied load in Figure 2.

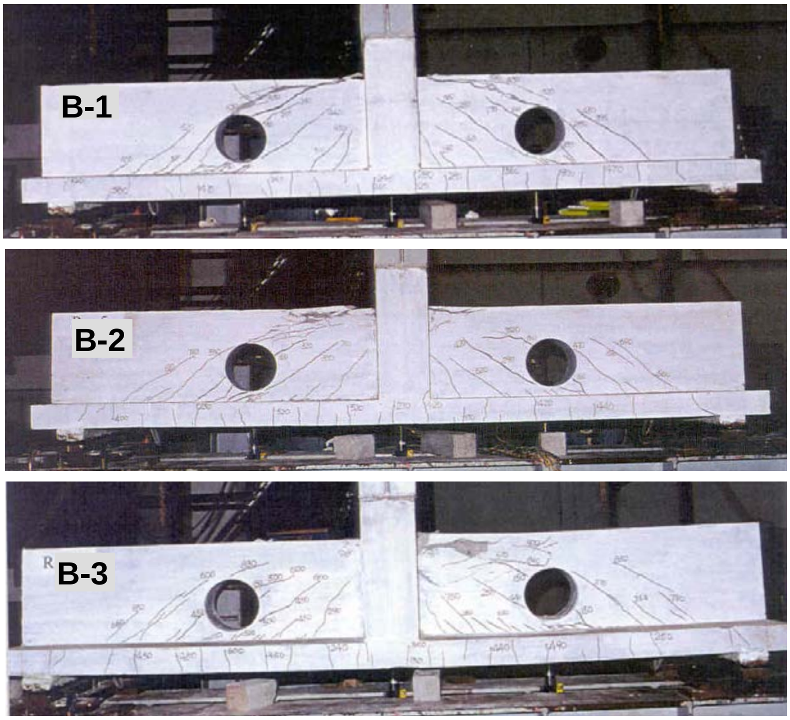


Figure 3: Cracking patterns of beams.

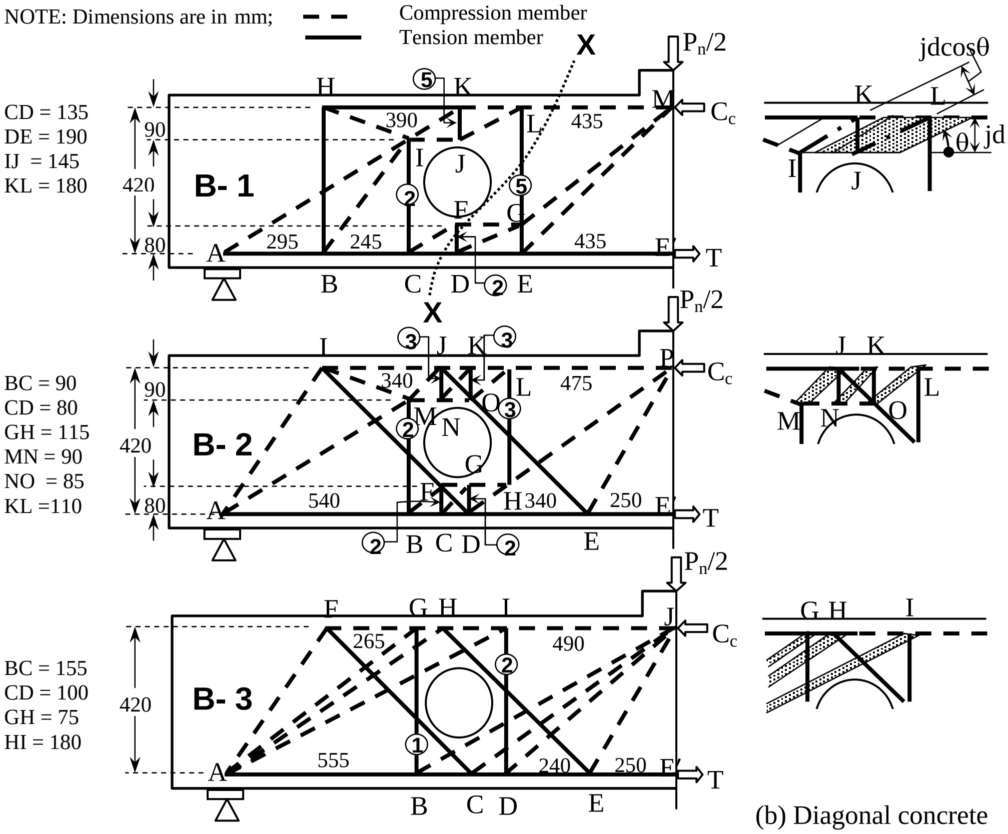
It may be seen that for the beam B-1, which did not have any diagonal bars, the principal compressive strain kept increasing dramatically from a load level of about 550 kN. This is in contrast with the response of beams B-2 and B-3 where it increased linearly until failure. It may also be noted from the results of B-2 and B-3 that at a given load level, the magnitude of the principal compressive strain becomes smaller as more amount of diagonal reinforcement is provided. Thus it seems that the provision of diagonal reinforcement helps reducing the compressive stress at the throat section of a beam with openings. The cracking patterns of the beams after failure are shown in Figure 3.

4.0 Analysis by Strut-and-Tie Model

In this analysis, it is considered that the applied loads on the beam are transmitted to the support by means of a system of tension ties and compressive struts, provided by the steel reinforcement and concrete, respectively, and they are interconnected at the nodes. Such a truss model assumes or requires that,

a. Forces in the truss members are in equilibrium.
b. Concrete resists only compression and has an effective compressive strength f_{ce} equal to $v\, f_c'$, where f_c' is the cylinder compressive strength of concrete, and v is the effectiveness factor. The value of v is usually less than 1.0.
c. Steel reinforcement is required to resist all tensile forces.
d. The centroidal axis of each truss member and the lines of action of all externally applied loads at a joint must meet at the nodes.
e. Failure occurs when a concrete compressive strut crushes, or when a sufficient number of steel tension ties yields to produce a mechanism.

Three typical truss models were developed, one each for beams B-1, B-2 and B-3. These beams contained only vertical stirrups, a combination of vertical stirrups and diagonal bars, and only diagonal bars, respectively as shear reinforcement around the opening (see Figure 1). Figure 4 shows the truss models drawn for half of the beams



(a) Truss geometry and dimensions.

(b) Diagonal concrete struts at throat

Figure 4: Truss models for the beams with a circular opening.

taking advantage of symmetry. The solid and dotted lines represent the tension ties and compression struts, respectively.

The trusses shown in Figure 4 have been modelled using both the steel strain readings and the cracking patterns shown in Figure 3. The number written beside the vertical tie members refer to the number of yielded stirrups, as determined from the strain readings. These vertical ties were placed according to the centroid of the yielded stirrups. At ultimate load, the diagonal reinforcement in both the beams B-2 and B-3 yielded.

4.1 *Load carrying capacity*

After the number of yielded bars has been identified and the forces in the corresponding tie members calculated, the forces in other truss members and hence the ultimate load carrying capacity of each beam can be determined by considering the equilibrium of forces at the nodes. In beam B-1 [Figure 4(a)], it may be easily shown by considering vertical equilibrium of the part of the truss to the left of section *X*-*X* that the applied load is twice the sum of the forces in the vertical ties *FD* and *LG*. Since *FD* consists of two 10 mm diameter two-legged stirrups, each with a tensile capacity of 48kN, and *LG* had five such stirrups, the nominal load carrying capacity, P_n, can be calculated as $2\ (2 \times 48 + 5 \times 48) = 672$kN.

Similarly, for beam B-2, the tensile capacity of each of the diagonals *ID* and *JE* inclined at an angle of 45^o to the beam axis is 141 kN (two T13 bars). Here the load is transferred through four load paths: *PHLOKNJMA*, *PDGCFBMA*, *PEJMA*, and *PDIA*. Hence, the value of P_n is twice the sum of the forces in LH, GD and the vertical component of the forces in *JE* and *ID*, that is, $P_n = 2\ (3 \times 48 + 2 \times 48 + 141 \sin 45^o + 141 \sin 45^o) = 880$ kN. In beam B-3, the tensile capacity of each of the diagonals *HE* and *FC* (each consisting of two T16 bars) is about 212 kN. Hence, the load carrying capacity $P_n = 2\ (2 \times 48 + 1 \times 48 + 212 \sin 45^o + 212 \sin 45^o) = 888$kN.

4.2 *Effective concrete strength*

The compressive strength of concrete in the strut members is affected by the extent of cracking. Various sources (Collins and Mitchell 1980; Rogowsky and MacGregor 1986; MacGregor 1998) give different effective strength factors to account for cracking. Some of the major parameters affecting the compressive strength of concrete are the gross tensile strains perpendicular to the axis of the strut member and the direction of cracking. In beam R-4, small cracks were observed near the top of the beam at the high-moment end of the opening. These cracks were not all parallel to the axis of the struts. Therefore the effective concrete strength in the compression chord would decrease. For the purpose of analysis, an effectiveness factor of 0.8 is applied to the concrete struts in the chord members, and 1.0 for the remaining struts in the solid section.

4.3 Ultimate load and failure mode

The forces calculated in each member on the basis of the derived ultimate load are presented in Table 2. The longitudinal steel bars at the bottom of the beams consisted of four 20 mm diameter and six 16 mm diameter high strength steel bars (4T20 and 6T16) with a total tensile capacity of 1340kN.

Table 2: Forces in idealised truss members for the beams at ultimate load.

B-1				B-2				B-3			
Member	Force	Hor. member	Force	Member	Force	Hor. member	Force	Member	Force	Hor. member	Force
AI	-634	FG	-160	AI	-122	FG	-107	AF	-182	FG	-254
HI	-281	IJ	-471	AM	-653	GH	-203	AG	-79	GH	-317
BI	-120	KL	-136	**MJ**	**-345**	NO	-176	**AH**	**-270**	HI	-691
CF	-186	LM	-607	**NK**	**-199**	NM	-313	**AI**	**-208**	IJ	-876
DG	-246	**C_c**	**-1029**	**OL**	**-227**	IJ	-169	BJ	-98	**C_c**	**-1372**
IK	**-466**	HK	264	BF	-144	JK	-513	CJ	-258	AB	576
JL	**-529**	AB	538	CG	-136	KL	-650	DJ	-147	BC	661
GM	-489	BC	610	DH	-337	LP	-826	EJ	-174	CD	1021
EM	-50	CD	770	HP	-584	**C_c**	**-1360**			DE	1133
		DE	996	EP	-116	AB	626			**T=EE'**	**1372**
		T=EE'	**1032**			BC	733				
						CD	829				
						DE	1203				
						T=EE'	**1362**				

Note: Positive values indicate tension force and negative values indicate compression. Force unit: kN

In beam B-1 [Figure 4(b)], struts *LJ* and *KI* at the throat section are the most critical ones. They are inclined to the beam axis at 27^o and 31^o, respectively. Thus the diagonal compression force in *LJ* is 5 × 48 / sin27^o = 529kN. The required width to resist this force, assuming an effective concrete strength $f_{ce}= 0.8 f_c'$, is 529,000 / (0.8 × 33.6 ×200) = 98.4mm. In a similar manner, the required width for strut *KI* is found to be 86.7 mm. To accommodate these two struts in the compression chord, the required width between *LJ* and *KI* is (98.4 +86.7) /2 = 92.6mm. But, as shown in Figure 4(b), the available width is *jd* cosθ = 90 × cos27^o = 80mm. It is obvious that the areas of the struts *LJ* and *KI* are overlapped or, in the other words, the compressive stress in the struts exceeds the assumed effective strength of 0.8 f_c'. At a load of 672 kN, the tensile force in tie EE′ is 1032 kN, which is less than the tensile capacity of the bottom longitudinal bars of 1340kN. Therefore, it may be concluded that the beam would have failed at this load by concrete crushing in strut *LJ*, as observed during the experiment.

For beam B-2, the width available was sufficient to accommodate the struts in the compression chord, as shown in Figure 4(b). At a loading of 880kN, the tensile force in tie *EE′* is 1362 kN, which exceeds the tensile capacity of the bottom longitudinal bars. Thus, the failure mode can be identified as flexural failure.

Similarly, Figure 4 (b) shows that in beam B-3 there is no problem of high stress in the compression chord at the ultimate load of 888 kN. At this load, the tensile force in tie *EE′* is 1372kN, which exceeds the tensile capacity of the bottom longitudinal bars leading to a flexural failure.

From the truss model constructed for beam B-1, it can be found that 5/7 or 71% of the total shear force (that is, 240 kN out of a total of 336 kN) was transferred from point *L* to *A* through the compression chord. This caused the high stress in the compression chord and led to the crushing of the diagonal concrete strut there. On the other hand, despite the compression chord in beam B-2 also carried a high shear force (59% of the total shear force or 260 kN), concrete did not crush at that location. Out of this shear force, 100 kN was transferred by the diagonal bars *JE* and the rest by vertical stirrups. This has helped to reduce the stress in the compression chord. Similarly, in beam B-3, the compression chord carried a shear force of 246 kN, of which 150 kN was transferred by the diagonal bars. The compression chord appeared to be quite safe from the standpoint of crushing of concrete strut. Thus for beams with openings under high shear force, the high compressive stress condition in the compression chord member can be effectively reduced by using diagonal reinforcement. It also helps in effective control of crack widths, as can be seen in Table 1.

Table 3: Ultimate shear strength and comparison with theoretical predictions

Beam	Experimental			Truss model		
	Collapse load P_e (kN)	Mode of failure	Proportion of shear force carried by compression chord	Load carrying capacity (kN)	Mode of failure	Proportion of shear force carried by compression chord
B-1	669.8	WC	67%	672	WC	71%
B-2	935.8	F	55%	880	F	59%
B-3	921.2	F	47%	888	F	55%

Note: WC is Failure by crushing of the concrete in the compression chord; F is Flexural failure

In Table 3, the ultimate loads and the modes of failure of the beams predicted by the truss model are compared with experimental results. The truss model gave a very good prediction of the ultimate load as well as the mode of failure. Furthermore, it predicts the proportion of total shear carried by the compression chord with a good degree of accuracy.

5.0 Conclusions

Three full-size T-beams tested previously by the authors have been analysed in this paper by using the familiar strut-and-tie model. The results of the analysis and a comparison with the test data reveal that the strut-and-tie model provides an excellent tool for predicting the ultimate strength of a beam that contains a discontinuity in the form of a transverse circular opening in the web. It also successfully allows for an explanation for the modes of failure observed in the test and identifies the role of diagonal reinforcement provided by the side of an opening. These findings could lead to a better and more efficient design of the disturbed opening region of a beam.

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