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PROBABILITY-BASED ANALYSIS OF SLOPE STABILITY

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Abstract: The aim of this paper is to present a probability analysis using the Monte Carlo simulation method of uncertainty (MCSM). The results of this method will be compared to all recognized method of slope stability such as Bishop simplified, Fellenuis, Janbu simplified and corrected, Spencer and Lowe-Karafiath which are in general in limit equilibrium. This study has been done by a normal frequency distribution relative for all the parameters taken into considerations. From the mean values and the standard deviations of the pore water pressure, cohesion and the internal angle of friction with the correlation relation between these parameters, a set of random values of pore water pressure, cohesion and internal angle of friction were generated by computing a Critical Probabilistic Slip Surface. The analysis of the obtained results indicates that the failure probability is affected by the standard deviation of the pore water pressure, cohesion, internal angle of friction and correlation coefficient. However, all methods of equilibrium limit are affecting the failure probability by taking into account one of these parameters following each case. Nevertheless, the probability of failure is not significantly affected by the standard deviation of the unit weight for all methods.

Keywords: *Probability, slope stability, Monte Carlo simulation, Latin Hypercube, factor of safety.*

1.0 Introduction

Conventional slope stability methods compute the factor of safety of earth or rock slope based only on a fixed set conditions or a given data which include shear strength parameters, pore-water pressure and slope geometry. This analysis searches for a minimum factor of safety using a trial and error procedure and is referred to as a deterministic approach. If the factor of safety is greater than unity (i.e. $F > 1$); the slope is assumed to be stable. On the contrary, if the factor of safety is less than unity (i.e. $F < 1$), the slope is assumed to be unstable or susceptible to failure. The analysis of an engineering structure involves consideration of the relationship between resistance or capacity, R, and load or demand Q. the factor of safety may be defined as the ratio, $F=R/Q$ and the safety margin as, $SM=(R-Q)$. In a probabilistic analysis, one or both of R

and *Q* may be regarded as random variables, each with a probability distribution, rather than as constants or single-valued parameters, as in deterministic analysis.

Uncertainties is associated with the calculated value of *F*, the conventional or deterministic safety factor which is, after all, based on single values of the input variables such as shear strength parameters and pore-water pressure. There are three primary sources of geotechnical uncertainties: (a) inherent variability, (b) measurement uncertainties, and (c) transformation uncertainties. The first results primarily from the natural geologic processes that produced and continually modify the soil mass in-situ. The second is caused by equipment, procedural and/or operator, and random testing effects. In general, tests that are highly operator-dependent and have complicated test procedures will have greater variability than those with simple procedures and little operator dependency, as describe d in detail elsewhere world (Kulhawy and Trautman, 1996). Random testing error refers to the remaining scatter in the test results that is not assignable to specific testing parameters and is not caused by inherent soil variability. The third component of uncertainty is introduced when field or laboratory measurements are transformed in to design soil properties using empirical or other correlation models (e.g., correlating the standard penetration test *N* value with the untrained shear strength).

Neglecting uncertainties in slope analysis is an important limitation of the conventional deterministic approach. In consequence, the conventional 'factor of safety' is often not a reliable indicator of slope performance. A probabilistic approach, on the other hand, allows for the systematic analysis of uncertainties and for their inclusion strength parameters and pore-water pressures may be regarded as random variables, each with a probability distribution, rather than deterministic values or constants. Consequently, the factor of safety *F* of a slope under specified condition must also be regarded as a random variable with a probability distribution. The terms 'reliability index' and 'probability of failure 'or' the probability of inadequate performance' were first introduced as performance indicators within a probabilistic framework. Evaluating these indicators complements the evaluation of a conventional factor of safety and enhances the assessment of slope reliability.

2.0 Objectives of Present Work

This study focusing on the influence of parameterizing probability standard deviation of cohesion, angle of internal friction, pore-water pressure, weight, and correlation coefficient between cohesion and angle of internal friction on the various method of slope stability such as Bishop simplified, Fellenuis, Janbu simplified and corrected, Spencer and Lowe-Karafiath which are general in limit equilibrium, and to base the interpretation of the results on the values of probability of failure and reliability index.

The simple homogeneous slope with geometry is presented in Figure 1. The soil properties are shown in Table 1.

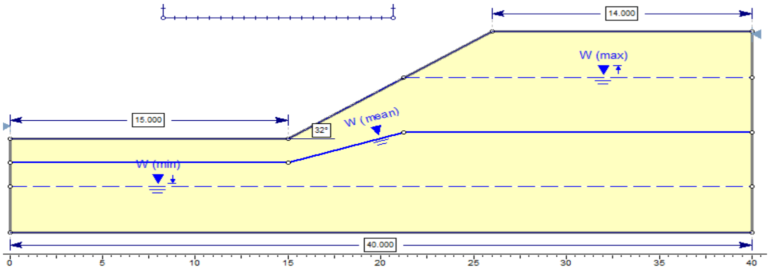


Figure 1: Cross-section

Table 1: Soil properties

Soil parameter	*Distribution*	*Mean*
Cohesion	Normal	5
Friction angle	Normal	22
Unit Weight	Normal	18

3.0 The Reliability Index

Reliability of slope stability is frequently measured by a "reliability index" β, or slope failure probability, *Pf*, which is defined as the probability that the minimum factor of safety (*Fs*) is less than unity (i.e., *Pf* =P (Fs<1)). Various solution method have been proposed to estimate β and (or) *Pf* (D'Andrea and Sangrey, 1982; Chowdhury and Xu, 1995 & Lacasse and Nadim, 1996). Among the most widely used methods are the First order second moment method (Christian *et al*., 1994; Hassan and Wolff, 1999), the First order reliability method (Low *et al*., 1997, 1998 & Low, 2005) and direct Monte Carlo simulation method (El-Ramly *et al*., 2002 & Griffiths and Fenton, 2004). For reference, Table 2 lists β and *Pf* to represent geotechnical components and systems and their expected performance level. The value of β commonly ranged from 1 to 5, corresponding to *Pf* varying from about 0.16 to $3x10^{-7}$. Geotechnical designs require a β value of at least 2 (i.e. *Pf*<0.023) for an expected performance level better than "poor". A reliability small *Pf* value (information about the tail of the probability distribution) is of great interest to geotechnical practitioners. This calls for reliability solution methods that can efficiently provide high-resolution information at the tail of the probability distribution (i.e. at relatively small probability levels).

Table 2: Reliability Index

Reliability index, β	*Probability of failure $Pf=\Phi(-\beta)$*	*Expected performance level*
1.0	0.16	Hazardous
1.5	0.07	Unsatisfactory
2.0	0.023	Poor
2.5	0.006	Below average
3.0	0.01	Above average
4	0.00003	Good
5	0.0000003	High

4.0 Monte Carlo Simulation

The Monte Carlo method was developed in 1949 when John von Neumann and Stanislav Ulam published a paper, "The Monte Carlo Method." The Neumann and Ulam concept specifically designated the use of random sampling procedures for treating deterministic mathematical situations. The foundation of the Monte Carlo gained significance with the development of computers to automate the laborious calculations. Figure 2 illustrates a general schematic for a Monte Carlo simulation (Hutchinson & Bandalos, 1997). The first step of a Monte Carlo simulation is to identify a deterministic model where multiple input variables are used to estimate a single value outcome. Step two requires that all variables or parameters be identified. Next, the probability distribution for each independent variable is established for the simulation model, (i.e. Normal, Beta, Log-normal, etc.). Next, a random trial process is initiated to establish a probability distribution function for the deterministic situation being modeled. During each pass, a random value from the distribution function for each parameter is selected and entered into the calculation. Numerous solutions are obtained by making multiple passes through the program to obtain a solution for each pass. The appropriate number of passes for an analysis is a function of the number of input parameters, the complexity of the modeled situation, and the desired precision of the output. The final result of a Monte Carlo simulation is a probability distribution and reliability index of the output parameter.

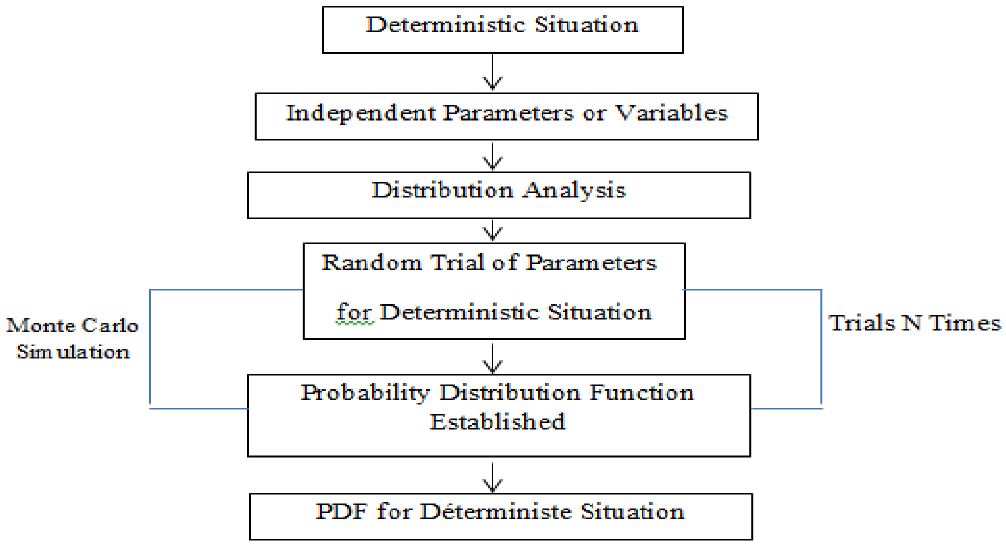
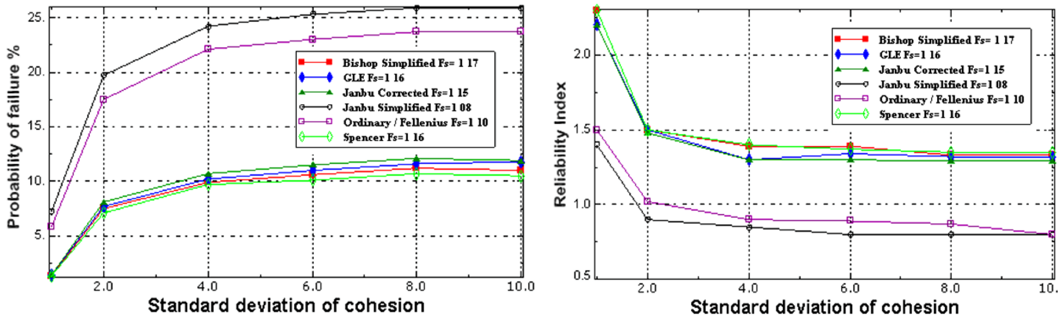


Figure 2: General Monte Carlo Simulation Approach (Hutchinson and Bandalos, 1997).

5.0 Results of the Probabilistic Analysis

5.1 Effect of Standard Deviation of the Cohesion

Figure 3a indicates that both ordinary/Fellenuis and Janbu results are much larger than the rest. By looking closer at the given results, we can see that the standard deviations of cohesion values are greater when compared to the rest of the methods. Once the standard deviation of cohesion values is less than 4.0, all the values increases in a linear fashion. Yet, when the value is greater than 4.0, the curve of all methods is leaning towards stabilization. The effect of small standard deviation of cohesion on reliability index has a less impact on the curve when the standard deviation is greater than 4 as depicted in Figure 3b.

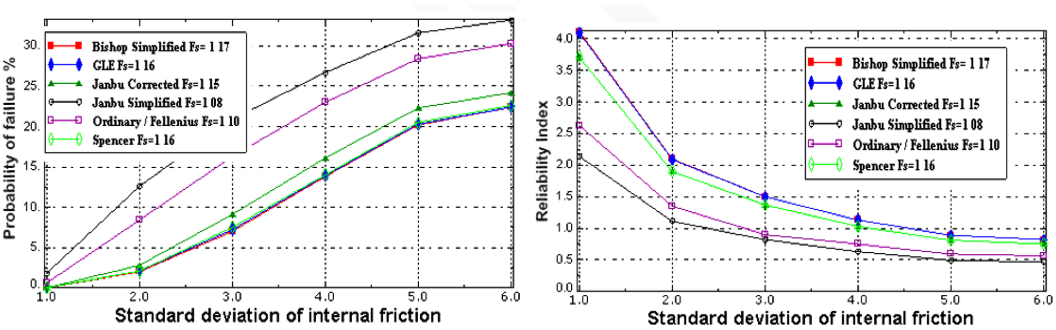


a) b)

Figure 3: a) The probability failure and b) Reliability index of the factor of safety for various standard deviation of the cohesion.

5.2 Effect of the Standard Deviation of the Angle of Internal Friction

Figure 4 shows that both ordinary/Fellenuis and Janbu methods give a greater probability of failure values than the other methods; the rest of the methods are almost identical. The highest probability of failure is recorded by Janbu simplified method at 35%.



(a) (b)

Figure 4: a) The probability failure and b) Reliability index of the factor of safety for various standard deviation of the angle of internal friction.

5.3 Effect of the Standard Deviation of the Weight

Figure 4 indicates the probability of failure is not affected by the standard deviation of the unit weight. The recorded failure probability remains almost identical as the standard deviation increases and this is true for all methods.

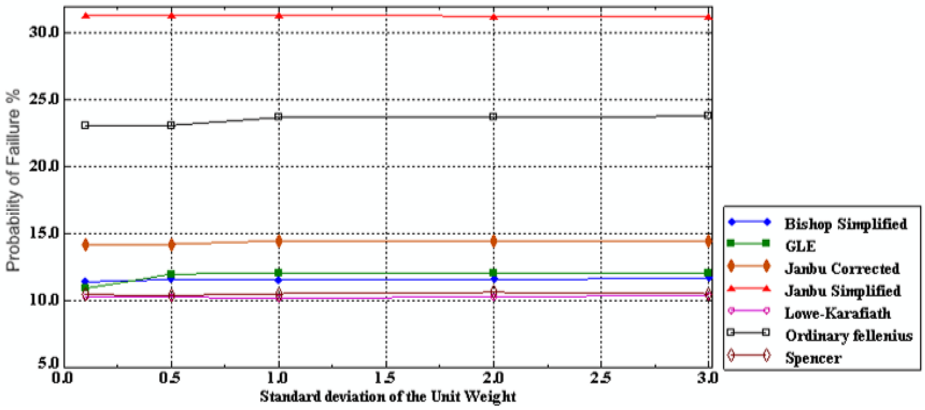
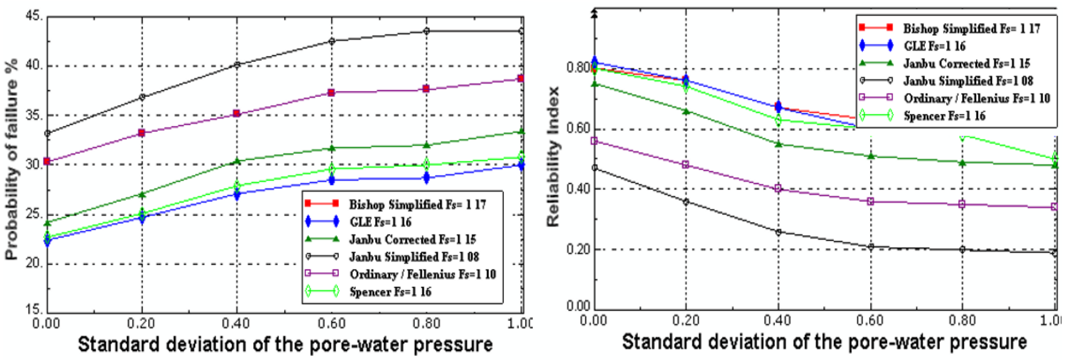


Figure 5: The probability failure and for various standard deviation of Weight.

5.4 *Effect of the Standard Deviation of the Pore-Water Pressure*

Figure 6 shows that the probability of failure experience huge increase when the standard deviation of pore water pressure increases from zero to 0.4. Beyond 0.4 values, the increment of probability of failure is considered marginal.

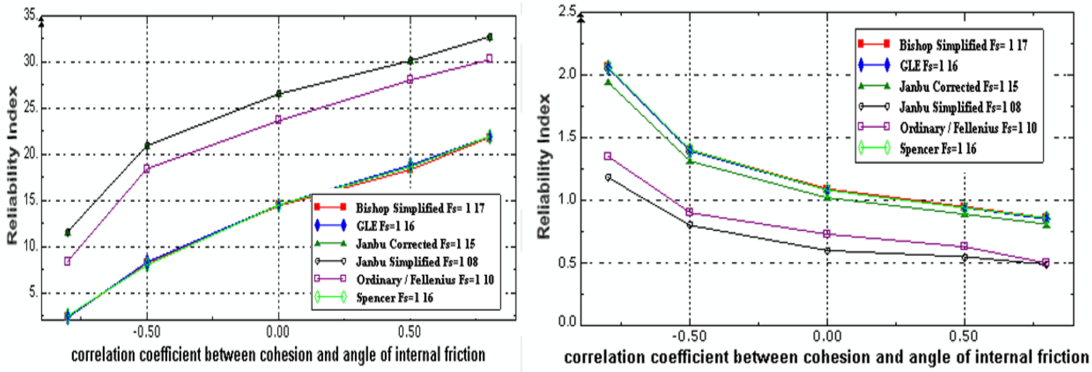


(a) (b)

Figure 6: a) The probability failure and b) Reliability index of the factor of safety for various standard deviation of pore-water pressure.

5.5 *Effect of the Correlation Coefficient between Cohesion and Angle of Internal Friction*

Figure 7 shows that the probability of failure and reliability index is greatly impacted by the increase of correlation coefficient between cohesion and angle of internal friction. The highest probability of failure is recorded at 33% by Janbu simplified method.



(a) (b)

Figure 7: a) The probability failure and b) Reliability index of the factor of safety for correlation coefficient between cohesion and angle of internal friction.

6.0 Conclusions

From the parametric study, the following observation can be made.

a) The results from the probability approach and reliability index are significantly affected by small magnitude of standard of the cohesion.

b) The probability approach and reliability index are affected by the magnitude of the angle of internal friction, correlation coefficient between the cohesion and the angle of internal friction. Meanwhile, the probability of failure and reliability index is not affected by the magnitude of standard of the weight.

c) Janbu simplified and Ordinary/Fellenius methods results gives a greater values in probability of failure and reliability index compared to the other methods especially in standard deviation of cohesion and correlation coefficient between cohesion and angle of internal friction.

References

Chowdhury, R. N., and Xu, D. W.(1995). “Geotechnical system reliability of slopes.” *Reliab. Eng. Syst. Saf.*, 47, 141–151Tang et al.1976.

Christian, J. T., Ladd, C. C., and Baecher, G. B.(1994). “Reliability applied to slope stability analysis.” *J. Geotech. Engrg*., 120(12), 2180–2207.

D’Andrea R. A. and Sangrey D. A. (1982) Safety factors for probabilistic slope design. *J Geotech Eng, ASCE*, 108(9):1108–1118, 1982.

El-Ramly, H., Morgenstern, N. R., and Cruden, D. M.(2002). “Probabilistic slope stability analysis for practice.” *Can. Geotech. J*., 39, 665– 683.

Griffiths, D. V., and Fenton, G. A.2004. “Probabilistic slope stability analysis by finite elements.” *J. Geotech. Geoenviron. Eng*., 130(5), 507–518.

Hassan, A. M. and Wolff, T. F. (1999). “Search algorithm for minimum reliability index of earth slopes.” J. Geotech. Geoenviron. Eng., 125 (4), 301–308.

Hutchinson, S. & Bandalos, D. (1997). A Guide to Monte Carlo Simulation Research for Applied Researchers. *Journal of Vocational Education Research*, 22, 233-245.

Kulhawy, F. and Trautman, C.H. (1996), “Estimation of in situ test uncertainty,” in Uncertainty in the Geologic Environment: From Theory to Practice, *Proceeding of Uncertainty ’96*, ASCE Geotechnical Special Publication No. 58.

Lacasse, S. and Nadim, F. (1996). “Uncertainties in characterizing soil properties.” Uncertainty in geologic environment: from theory to practice, Proc., Uncertainty ’96, *Geotechnical Special Publication* No. 58, C. D. Shackelford and P. P. Nelson, eds., Vol. 1, ASCE, New York, 49–75.

Low, B. K. (2005). Reliability-based design applied to retaining walls. *Géotechnique*, 55(1),63–75.

Low, B. K., and Tang, W. H. (1997). “Efficient reliability evaluation using spreadsheet.” J. of Engrg. Mech., ASCE, New York, 123(7), 749-752.

Low, B. K., Gilbert, R. B., and Wright, S. G. (1998). “Slope reliability analysis using generalized method of slices.” *J. of Geotech. & Geoenvironmental Engrg.*,